Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2021

Subject Code:3140708 **Date:24/12/2021**

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Show that for any two sets A and B, $A-(A\cap B)=A-B$. **03**

(b) If $S=\{a,\ b,\ c\}$, find nonempty disjoint sets A_1 and A_2 such that $A_1\cup A_2=S$. Find the other solutions to this problem. **04**

(c) Using truth table state whether each of the following implication is tautology. **07**

a) $(p\wedge r)\rightarrow p$

b) $(p\wedge q)\rightarrow(p\rightarrow q)$

c) $p\rightarrow(p\vee q)$

Q-2 **(a)** Given $S=\{1,2,3,----,10\}$ and a relation R on S. Where, $R=\{\langle x,y\rangle|x+y=10\}$. What are the properties of relation R? **03**

(b) Let L denotes the relation "less than or equal to" and D denotes the relation "divides". Where xDy means "x divides y". Both L and D are defined on the set $\{1,2,3,\ 6\}$. Write L and D as sets, find $L\cap D$. **04**

(c) Let $X=\{1,2,3,4,5,6,7\}$ and $R=\{\langle x,y\rangle|x-y\ is\ divisible\ by\ 3\}$. Show that R is an equivalence relation on. Draw the graph of R. **07**

OR

(b) Define equivalence class generated by an element $x\in X$. Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by **07**

$$R=\{\langle x,y\rangle|x\in Z\wedge y\in Z\wedge(x-y)\ is\ divisible\ by\ 3\}$$

Determine the equivalences classes generated by the element of Z.

Q.3 **(a)** Let $f(x)$ be any real valued function. Show that $g(x)=\frac{f(x)+f(-x)}{2}$ is always an even function where as $h(x)=\frac{f(x)-f(-x)}{2}$ is always an odd function. **03**

(b) The Indian cricket team consist of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can cricket eleven be selected if we have select 1 wicket keeper and at least 4 bowlers? **04**

(c) Let A be the set of factors of particular positive integer m and $\leq$ be the relation divides, that is **07**

$\leq=\{\langle x,y\rangle|x\in A\wedge y\in A\wedge(x\ divides\ y)\}$ Draw the Hasse diagrams for

a) $m=45$

b) $m=210$.

OR

Q-3 **(a)** Find the composition of two functions $f(x)=e^x$ and $g(x)=x^3$, $(f\circ g)(x)$ and $(g\circ f)(x)$. Hence, show that $(f\circ g)(x)\neq(g\circ f)(x)$. **07**

(b) In a box, there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen? **04**

(c) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hass diagram for $\langle\rho(A),\subseteq\rangle$ for **07**

a) $A=\{a,b,c\}$

b) $A=\{a,b,c,d\}$

Q.4 **(a)** Let $\langle L, \leq \rangle$ be a lattice. Show that for $a, b, c \in L$, following inequalities holds. **07**

$$a \oplus (b \star c) = (a \oplus b) \star (a \oplus c)$$
$$a * (b \oplus c) = (a * b) \oplus (a * c)$$

(b) Let $G = \{(a, b) | a, b \in R\}$. Define binary operation $(*)$ on G as $(a, b), (c, d) \in G, (a, b) * (c, d) = (ac, bc + d)$. Show that an algebraic structure $(G, *)$ is a group. **07**

OR

Q-4 **(a)** Let G be the set of non-zero real numbers. Define binary operation $(*)$ on G as $a * b = \frac{ab}{2}$. Show that an algebraic structure $(G, *)$ is an abelian group. **07**

(b) Explain the following terms with proper illustrations. **07**

a) Directed graphs
b) Simple and elementary path
c) Reachability of a vertex
d) Connected graph.

Q-5 **(a)** Show that sum of in-degrees of all the nodes of simple digraph is equal to the sum of out-degrees of all the nodes and this sum equal to the number of edges in it. **07**

(b) Let $= \{1, 2, 3, 4\}$. For the relation R whose matrix is given, find the matrix of the transitive closure by using Warshall's algorithm. **07**

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

OR

Q-5 **(a)** Define tree and root. Also prove that tree with n vertices has $n - 1$ edges. **07**

(b) Define in-degree and out-degree of a vertex and matrix of a relation. Let $A = \{a, b, c, d\}$ and let R be the relation on A that has the matrix **07**

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

****